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MAX-MIN EDGE MAGIC AND ANTIMAGIC LABELING

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Abstract:
In this paper we introduce a new type of edge magic and antimagic labeling and study the same for various classes of graphs.

Key Words: Graph, labeling, function, magic, antimagic

Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions .Labeled graphs are becoming an increasingly useful family of Mathematical Models from abroad range of applications. The graph labeling problem that appears in graph theory has a fast development recently. This problem was first introduced by Alex Rosa in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallography, coding theory, radar, astronomy, circuit design, and design of good Radar Type Codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design. An enormous body of literature has grown around the subject in about 1500 papers. They gave birth to families of graphs with attractive names such as graceful, Harmonious, felicitous, elegant, cordial, magic antimagic, bimagic and prime labeling etc. A useful survey to know about the numerous graph labeling methods is the one by J.A. Gallian recently [2012][1].All graphs considered here are finite simple and undirected.

The vertex-weight of a vertex v in G under an edge labeling is the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to the entire edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or Vertex-magic total labeling respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling. The edge-weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e under a total labeling, we also add the label of e. Using edge-weight, we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

Definition 1.1: A (p, q) graph G is said to be $(1,0)$ edge-magic with the common edge count k if there exists a bijection $f : V(G) \to \{1, \ldots, p\}$ such that for all $e = (u,v) \in E(G), f(u) + f(v) = k$. It is said to be $(1, 0)$ edge-antimagic if for all $e = (u,v) \in E(G), f(u) + f(v)$are distinct.

Definition 1.2: A (p,q) graph G is said to be $(0,1)$ vertex-magic with the common vertex count k if there exists a bijection f $: E(G) \to \{1, \ldots, q\}$ such that for each $u \in V(G), e \in\Sigma f(e) =k$ for all $e = (u,v) \in E(G)$ with $v \in V(G)$. It is said to be $(0, 1)$ vertex-antimagic if for each $u \in V (G), e \in\Sigma\ f(e)$ are distinct for all $e =(u,v) \in E(G)$with $v \in V(G)$.

Definition 1.3: A (p,q) graph G is said to be $(1,1)$ edge-magic with the common edge count k if there exists a bijection $f : V(G) \cup E(G) \to \{1, \ldots, p+q\}$ such that $f(u) + f(v) +f(e) = k$ for all $e = (u,v) \in E(G)$. It is said to be $(1,1)$ edge-antimagic if $f(u) + f(v) + f(e)$ are distinct for all $e = (u,v) \in E(G)$.

In this paper we introduce a new type of edge magic and antimagic labeling and study the same for various classes of graphs.

Main Results

Definition 2.1: Let G(V,E) be a simple graph with *p* vertices and *q* edges. An injective function *f : V(G) → N,* is said to be max-min edge magic labeling if for every edge uv in E, max{f(u),f(v)}/min{f(u),f(v)} is a constant where *N* is the set of natural numbers. *f* is said to be an max-min edge antimagic labeling if max{f(u),f(v)}/min{f(u),f(v)} is distinct.

Theorem 2. 2: P_n admits max-min edge magic labeling.

Proof: Let the vertices V={$v_1,v_2,\dots,v_n$ } and Let f: V→ {a^i}, a≥2, 1≤i≤n f(v_i)= a^i, such that for all xy €E,f(y)=af(y) for constant a.

We calculate the edge labels λ(uv) = max{f(u),f(v)}/min{f(u),f(v)}.

For1≤i≤n, λ (v_iv_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})} =a^{i+1}/a^i=a.

Hence P_n admits max-min edge magic labeling.

Theorem 2.3: P_n admits max-min edge antimagic labeling for all n.

Proof: Let the vertices V={$v_1,v_2,\dots,v_n$ } Let f:V→N such that f(v_i)= i;1≤i≤n, and the edge set E ={v_i v_{i+1}:1≤i≤n-1}.

For1≤i≤n-1, let λ (v_iv_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})}
= (i+1)/i,.

For 1≤ i,j≤n-1 clearly λ (v_i v_{i+1}) ≠ λ (v_j v_{j+1}) .

If λ (v_i v_{i+1}) = λ (v_j v_{j+1})

then i+1/i = j+1/j

⇒ ij+j=ij+i

⇒ i=j which is a contradiction. Hence all edges are distinct, proving the theorem.

Theorem 2.4: C_n admit admits max-min edge antimagic labeling for all n.

Proof: The cycle graph has n vertices and n edges. . Let the vertices V={$v_1,v_2,\dots,v_n$ } Let f:V→N such that f(v_i)= i;1≤i≤n, and

the edge set E = {v_i v_{i+1}:1≤i≤n-1} ∪ { v_1v_n}

for 1≤i≤n-1 , λ (v_iv_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})} = (i+1)/i ,

and λ (v_1v_n) = max{f(v_1),f(v_n)}/min{ f(v_1),f(v_n)}=n.

Since λ (v_iv_{i+1}) ≠ λ (v_1v_n),

If λ (v_i v_{i+1}) = g (v_1v_n),

then ⇒(i+1)/i = n

⇒ i+1 = ni which is a contradiction.

clearly all edges are distinct proving the theorem

Theorem 2.5: C_n^+admits admits max-min edge antimagic labeling.

Proof: Let the vertices V= {v_1, $v_2,\dots,v_{2n}$ } Let f: V→N such that

f(v_i)=2i -1;1≤i≤n,f(v_i)=2i;n+1≤i≤2n.

and the edge set E = { v_i v_{i+1}:1≤i≤n-1}∪ { v_1 v_n }∪ { v_i v_{i+1}: n+1≤i≤2n}

for 1≤i≤n-1, λ (v_iv_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})} = (2i+1)/ (2i-1).

also λ (v_1v_n) = max{f(v_1),f(v_n)}/min{ f(v_1),f(v_n)}=n.

and for n+1≤i≤2n, λ (v_iv_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})} = 2i+2/2i = i+1/i.

Clearly all edges are distinct proving C_n^+admits admits max-min edge antimagic labeling.

Theorem 2.6: The wheel graph admits max-min edge anti magic labeling.

Proof: The wheel graph has n vertices and 2(n-1) edges. Let the vertices V={$v_1,v_2,\dots,v_n$ } Let f:V→N such that f(v_1)= 1, f(v_i)= 2i-1;2≤i≤n-1, and the edge set E = { v_1v_i :2 ≤ i ≤n-1}∪ { v_iv_{i+1}: 2 ≤ i ≤n-1}∪ {v_2v_n}.

For 2 ≤ i ≤n-1, λ (v_1 v_i) = max{f(v_1),f(v_i)}/min{ f(v_1),f(v_i)} = i/1 =i,

For 2 ≤ i ≤n-1, λ (v_i v_{i+1}) = max{f(v_i),f(v_{i+1})}/min{ f(v_i),f(v_{i+1})}= i+1/(i),

and λ (v_2v_n) = max{f(v_2),f(v_n)}/min{ f(v_2),f(v_n)}= n/2.

Clearly all edges are distinct, proving the theorem.

Theorem 2. 7: $K_{1,n}$ admits max-min edge antimagic labeling .

Proof: Let the vertices V={$v_1,v_2,\dots,v_{n+1}$ }, and the edge set be E = { v_1 v_i: 2≤i≤n+1} now for 2≤i≤n+1, max{f(v_1),f(v_i)}/min{ f(v_1),f(v_i)}= i .Clearly all edges are distinct and also are in Arithmetic progression, hence : $K_{1,n}$ admits max-min edge antimagic labeling .

Theorem 2. 8: The comb graph admits max-min edge antimagic labeling.

Proof: Let the vertices V= {v_1, v_2,…,v_n ,u, u_2,…,u_n }Let f:V→N such that $f(v_i)= 2i-1;1\leq i\leq n$ and $f(u_i)= 2i;1\leq i\leq n$.Let E={ v_i u_i; $1\leq i\leq n$}∪ { v_i v_{i+1} $1\leq i\leq n-1$},

and also for $1\leq i\leq n$ λ (u_i v_i) = max{f(u_i),f(v_i)}/min{f(u_i),f(v_i)}= 2i/(2i-1) and for $1\leq i\leq n-1$ λ (v_i v_{i+1}) = max{f(v_i),f(v_{i+1})}/min{f(v_i),f(v_{i+1})}= 2i+1/(2i-1) are distinct.

Hence the comb graph admits max-min edge antimagic labeling.

Corollary: *binary tree, C^n_t (C_n), gear graph, C_n (C_n), corona ,Sunflower graph, one vertex union of 't' isomorphic copies of any cycle is max-min edge antimagic labeling.*

Conclusion

Max-Min edge antimagic labeling is studied for paths, cycle graph wheel graph, star graphs, *binary tree, C^n_t (C_n), gear graph, C_n (C_n), corona ,Sunflower graph, one vertex union of 't' isomorphic copies of any cycle* Also max-min edge magic labeling is observed for path graphs.Investigating max-min edge magic labeling for other classes is our future work.

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